FYS 3610 WEEK 40

EXERCISE 1

From the Maxwell's equations we have

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (2.1)$$

$$\nabla \cdot \vec{B} = 0 \qquad (2.2)$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} \qquad (2.3)$$

and the generalized Ohm's law is given by

$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B}) \qquad (2.4)$$

a) Show that the time varying magnetic field can be expressed as

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \vec{B} + \nabla \times (\vec{v} \times \vec{B}) \qquad (2.5)$$

(Hint: $\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B}$)

b) Infer the magnetic Reynolds number (R_m) by taking the ratio of the second term to the first term on the right hand side of Equation 2.5. Use L as a characteristic scale length for changes of the field and flow.

c) Discuss the usage of the Reynolds number as an indicator of whether the frozen-in-flux concept is valid or not. Why does the frozen-in-flux concept (ideal MHD) break down locally near a reconnection site.

EXERCISE 2

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EXERCISE 2

a) Sketch a vertical profile of electron density versus altitude for noon sunlit conditions, and also for midnight non-auroral conditions. Indicate D, E, and F-regions on your sketch.

b) Show that the ion and electron drifts perpendicular to the magnetic field can be written as:

$$\vec{v}_{i\perp} = \frac{k_i}{1+k_i^2}\frac{\vec{E}_\perp}{B} + \frac{k_i^2}{1+k_i^2}\frac{\vec{E}_\perp \times \vec{B}}{B^2} \qquad \text{Eq. 2.1}$$

$$\vec{v}_{e\perp} = -\frac{k_e}{1+k_e^2}\frac{\vec{E}_\perp}{B} + \frac{k_e^2}{1+k_e^2}\frac{\vec{E}_\perp \times \vec{B}}{B^2} \qquad \text{Eq. 2.2}$$

where $k_i = \frac{\Omega_i}{\nu_{in}}$ and $k_e = \frac{\Omega_e}{\nu_{en}}$. (The neutral wind has been neglected).

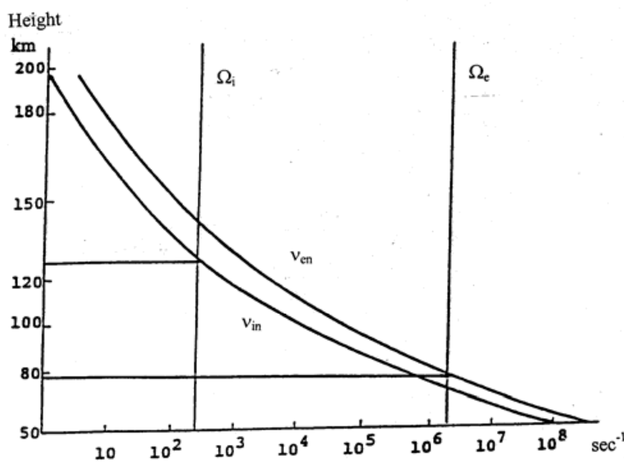

Figure 2.1

c) Given Figure 2.1 discuss the rotation of $\vec{v}_i$ and $\vec{v}_e$ by height as well as their variation in amplitude by height from 50 to 200 km.

d) Why is the current below 90 km negligible, and why is the current aboe 200 km zero.

Problem 3

THE SUN

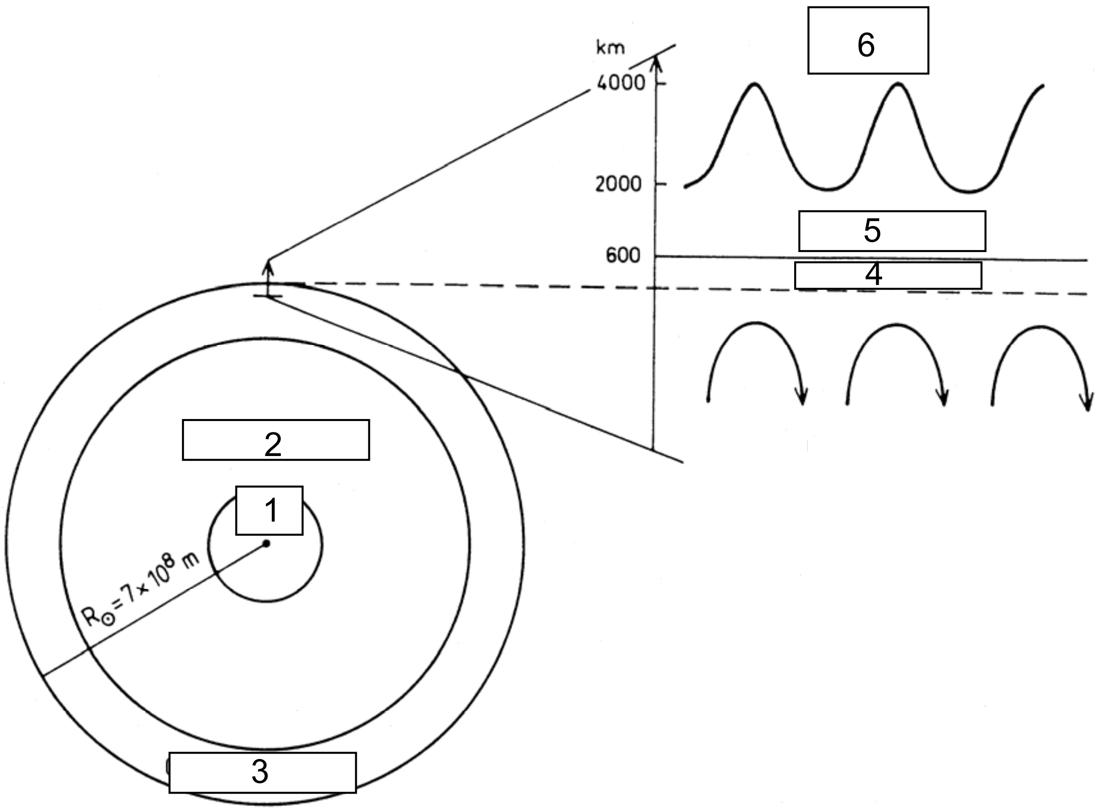

Figure 2

a) Name different regions 1- 6 in Figure 2.

b) Give a brief characteristics of sunspots (where do they occur, magnetic field, temperature,11-year cycle).

c) The solar radius is 6.960 x 10^5 km; the Sun weights 1.989 x 10^{30} kg. What is the escape velocity?

d) The Sun emits 3.9 x 10^{26} Js^{-1}. If all the energy emitted comes from fusion in the core, how much mass is burned off per second of the Sun? How long will it take to burn off 1% of the mass?

e) The total radiated power from the Sun is:

$$Q_S = 4\pi R_S^2 E_S = 3.9 \times 10^{26}\,\text{W}$$

Show that the radiated energy per unit area at 1 AU (1.496 x 10^8 km) is 1380 Wm^{-2}.